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SUPERSONIC TURNS WITHOUT SUPERBOOMS

by

H. S. Ribner

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SUMMARY

It is shown that focussed booms that arise in turning flight can be suppressed by the simple (although not always practicable) expedient of slowing down the aircraft. The correct deceleration will eliminate the local curvature of the wave front responsible for the focussing. Specifically, the tangential deceleration resolved along the normal to the wave front is adjusted to cancel out the centripetal acceleration similarly resolved.

Horizontal turns of a prescribed limiting sharpness are not of concern for this suppression technique: their focussed booms will be cut off from reaching the ground by atmospheric refraction. The minimum turn radius for focus cut-off is related herein in a simple fashion to the tabulated width of the sonic boom carpet for rectilinear flight, as a function of Mach number and altitude.

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INTRODUCTION

Maneuvers in supersonic flight may lead to superbooms; sonic booms much intensified by focussing and with enhanced potential for ground damage. The superbooms normally occur on the inside of a turn. In the present paper it is shown that the focussing effect can be suppressed by the simple expedient of slowing down the aircraft during the turn.

The underlying principles are brought out in Fig. 1. In the lefthand sketch a supersonic transport (SST) in straight and steady flight produces a cone-shaped bow wave (shock). This looks like a V on the paper: it resembles the bow wave of a boat. In fact, one can use the analogy with boat waves in visualizing the wave behaviour. In the middle sketch, if the SST makes a turn at constant speed the bow wave tries to follow and become unsymmetrically curved. The wave on the inside of the turn advances along the dotted lines (sound rays). The curvature of the wave tilts the rays and they converge to a focus. In the general region of the focus we have a magnified sonic boom: a superboom.

Look again at the lefthand sketch. If the SST slows down the nose of the bow wave is 'pushed in' and the bow wave becomes convex outward. This is the opposite of the concave curvature found inside the constant speed turn. So perhaps we should slow down while turning: then the two opposite curvature tendencies may cancel each other.

The righthand sketch - which is supported by analysis - shows that this is indeed the case. The cancellation of curvature is complete and the righthand wave is quite straight. The sound rays (in the plane of the paper) are straight and parallel, with no tendency toward a focus or consequent superboom.

The speed must not be reduced below the speed of sound during the turn; otherwise there will be a superboom on reaccelerating. This dictates the permissible angle of turn. Turn angles using this scheme are, in fact, severely curtailed at low supersonic speeds. But at these low speeds the major difficulty lies in the procedure calling for deceleration when, in fact, acceleration is required to get up to cruising speed.

The foregoing has been a simple qualitative account of the rationale for the slowdown maneuver. The ideas are developed quantitatively and in some depth in the main text. Additionally, the three-dimensional locus of focussed booms arising from horizontal turns is studied. The circumstances are examined wherein atmospheric refraction will bend the sound rays sufficiently to cut off the focussed booms from reaching the ground. A geometric argument is developed which relates the minimum turn radius for focus cutoff to the cutoff width of the sonic boom carpet in rectilinear flight.

RAO'S ANALYSIS

Rao (Ref. 1) has applied ray acoustics to the analysis of the effects of aircraft maneuvers on sonic boom intensities. The sound rays represent the trajectories of elements of the expanding wave front, and are drawn perpendicular to it. In straight and steady flight the wave is conical and the rays spread (in one plane) in proportion to distance s from the flight path.

The expansion ratio (normalized area) of a ray tube, which governs the boom intensity, is thus

$$E(s) = s \tag{1}$$

In accelerated flight - turning or rectilinear - this is generalized to (Ref. 1)

$$E(s) = s\left(1 - \frac{s}{\lambda}\right) \tag{2}$$

where λ is the outward radius of curvature of the bow wave in planes locally tangential to the flight path (λ is infinite for the straight line generators of a conical bow wave). The wave front curvature causes the rays to focus - to a line, not a point - at a distance $s = \lambda$; this is shown by the vanishing of the ray tube area, equation (2).

Rao showed that general maneuvers dictate the radius of curvature of the bow wave according to

$$\lambda = \frac{a_o^2 \, (M^2-1)}{a_{eff}} \tag{3}$$

with

$$a_{eff} = \frac{U^2}{R}\left(\frac{\sqrt{M^2-1}}{M}\cos\theta\right) + \frac{dU}{dt}\left(\frac{1}{M}\right) \tag{4}$$

The factors U^2/R and dU/dt are just the centripetal and axial accelerations of the aircraft, respectively; M is the flight Mach number U/a_o, and θ is the dihedral angle between the plane in which λ is measured and the plane of curvature, such that $\theta = 0$ on the inside of the turn.

It is easy to show that a_{eff} of Eq. (4) is an effective acceleration; it is the component of the resultant aircraft acceleration resolved along the sound ray emanating from the bow wave where λ is measured: the first term is the component of the centripetal acceleration and the second term is the component of the tangential acceleration.

When a_{eff} is zero, the radius of curvature λ is infinite and there is no focussing: Eq. (2) reduces to (1). This normally occurs when both terms of a_{eff} are zero: unaccelerated flight. But it can also occur when the two terms of a_{eff} are equal and opposite: the component of centripetal acceleration resolved along the sound ray is balanced by a component of tangential deceleration resolved along the ray. The latter possibility is exploited in this paper.

PHYSICAL INTERPRETATION

Consider an aircraft moving faster than sound along a curved path (Fig. 2). The circles represent sound waves that were emitted successively: their envelope is the bow wave. In rectilinear flight this envelope is conical, so that in the plane of the paper it resembles a V. But in the curved flight of Fig. 2 the envelope has a cusp on the inside of the turn. The tip of the cusp is a point of focus. This is brought out by the ray diagram of Fig. 3. The sound rays - orthogonal trajectories of points on the wave front (envelope) - proceed as shown. The rays converge to their own envelope (caustic) on the inner circle. The convergence is a condition of focus because the ray tube area

goes to zero; the caustic is thus a locus of focus booms in the plane of flight.

Figure 4 re-examines the early part of Fig. 2 at a certain time. The sound wave emitted from point O of the flight path has now reached point P of the bow wave. As time goes on this wave will continue to grow and its center will be left further behind by the receding aircraft. The point of tangency P will appear to run along the bow wave away from the aircraft. From the geometry of the figure the effective velocity of P is U cosμ.

The radius of curvature of the bow wave is Rao's λ (the dihedral angle θ = 0 for this plane). It is given by the usual kinematic relation

$$\lambda = \frac{(\text{effective velocity})^2}{\text{effective acceleration}} \qquad (5)$$

We have shown that the effective velocity of P is the component of aircraft velocity resolved along the ray: the quantity U cosμ. By a somewhat deeper argument Rao has shown that the effective acceleration of P is the component of aircraft acceleration resolved along the ray: the quantity a_{eff}. Putting these together allows us to recover equation (3), since $U \cos\mu = a_o\sqrt{M^2-1}$. Some of these ideas are illustrated by Fig. 5.

NO FOCUS CONDITION

Earlier arguments based on the physics of the situation express the condition for non-focussing of the boom signature in several equivalent ways: the bow wave radius of curvature λ in the plane under consideration (defined by θ) is infinite - the wave front is straight; the component of airplane resultant acceleration resolved along the sound ray in this plane (the effective acceleration a_{eff}) is zero, or the components of centripetal and tangential acceleration, so resolved, cancel. These conditions are exhibited in Fig. 6.

If the rate of turning of the aircraft path is dϕ/dt, this may replace U/R in equation (4). Then the no-focus condition $a_{eff} = 0$ yields

$$-\frac{dM}{dt} = M\sqrt{M^2-1}\ \frac{d\phi}{dt} \qquad (6)$$

This specifies the required rate of deceleration - dM/dt associated with the rate of turn dϕ/dt.

In terms of the Mach angle $\mu = \sin^{-1}(1/M)$ the terms in M combine to dμ/dt:

$$\frac{d\mu}{dt} = \frac{d\phi}{dt} \qquad (7)$$

$$\mu_2 - \mu_1 = \phi \qquad (8)$$

Thus the slowing down required by the no-focus condition increases the Mach angle μ by precisely the angle of turn ϕ. This clearly dictates an upper limit to ϕ for a given initial Mach number such that the final Mach number shall not be subsonic (μ_2 predicted > 90° by equation (8)).

An example of a no-focus 30° turn is shown in Fig. 7. The initial conditions are $M_1 = 2.00$, $\mu = 30^\circ$, and the deceleration yields the final conditions $M_2 = 1.15$, $\mu_2 = 60^\circ$. The change in μ matches the turn angle of 30°, as it should; moreover, the bow wave on the inside of the turn (in the plane of the turn) is straight.

Figure 8 shows the evolution of the bow waves of Fig. 7 as the envelope of sound waves emitted by the passage of the aircraft. The progressively decreasing separation of the centers of the sound waves (for equal times) reflects the deceleration of the aircraft. Notice how the bow wave envelope on the inside of the turn remains straight; hence the sound rays normal to the bow wave (not shown) must remain parallel and cannot converge to a focus.

LOCUS OF FOCUSSED BOOMS IN THREE DIMENSIONS

The examples have referred to booms that focus in the plane of the turn. We are primarily concerned, however, with booms that focus in or near the ground plane. Figure 9 shows how focussed booms propagate in three dimensions. If the speed of sound is uniform the locus of the focussed booms will be on a circular cylinder; the line of focus (caustic) will be a kind of spiral curve around the cylinder and will reach the ground.

This cylinder is the locus of all points in the rotating wave pattern moving at the speed of sound. In terms of the flight speed U, sound speed a, and flight path radius of curvature R, the radius of the Mach 1 cylinder is simply:

$$r = \frac{a}{U} R \qquad (9)$$

No wave envelopes (Mach waves) can penetrate inside the cylinder, since the motion of the pattern there is subsonic.

This concept of a Mach 1 cylinder can be extended to the real atomosphere by allowing for the increase of sound speed with decreasing altitude. Equation (9) still applies, with 'a' showing this increase. The effect is to flare the cylinder to larger radius at the ground (Fig. 9).

But in the real atmosphere, the caustic line of focussed booms spiraling down the cylinder may not reach the ground. In a wide range of circumstances the focus line will be cut off by refractive curvature of the sound rays. The cutoff mechanism is the same one that limits the width of the sonic boom carpet to the order of 50 to 60 miles for rectilinear flight.

This focussed boom cutoff has been explored in French studies, summarized in Ref. 2, by what are inferred to be detailed computer studies. However, we can approximate their numerical results by means of a very simple phenomenological model. In Fig. 10, B is the boom carpet half-width obtained from computed curves as a function of flight Mach number and altitude (e.g. Ref's. 3, 4*). A is the projection of the last ray of rectilinear flight drawn to the start of the focussed boom locus.

It is evident from the figure that when A is greater than B, the terminus of A will be outside the boom carpet. Thus the condition A = B is the cutoff

* The curves for a standard atmosphere are shown in Fig. 4 of Ref. 3 on an ordinate grid of 2.5 mile spacing and in Fig. 4 of Ref. 4 at reduced scale on an ordinate grid of 10 mile spacing.

condition for focussed booms. By geometry this is equivalent to

$$R \cos^2 \mu_g = B \tag{10}$$

Here R is the minimum radius of curvature of the flight path for which focussed booms will just reach the ground; for a radius larger than R the focus will be cut off. Correspondingly,

$$n'g = \frac{U^2}{R} = \frac{M^2 a^2}{R} \tag{11}$$

is the maximum centripetal acceleration that can be permitted without focussed booms reaching the ground, and

$$\tan\Phi = n' \tag{12}$$

refers to the corresponding maximum permissible airplane bank angle Φ.

Wanner et al in Ref.2, without specifying the computational details, have presented a chart of the limiting bank angle Φ (their Fig.4, reproduced herein as Fig. 11) for various altitudes and Mach numbers. With use of equations (11) and (12) we can immediately obtain therefrom the properties of the sharpest turns that can be negotiated without focus booms reaching the ground, namely, the maximum centripetal acceleration n' and the minimum turn radius R.

In Table I we have tabulated values of n' and R obtained in this way from Fig. 11 for a series of Mach numbers for flight at 11 km. (36,000 ft.) altitude in a standard atmosphere. Corresponding values of R calculated by the method of the present paper are likewise tabulated; these were obtained from the relation A = B of Fig. 10, utilizing values of B computed by Kane and Palmer (Ref.3).

It is seen that the agreement between the two sets of values of minimum turn radius R is quite good. The small discrepancies are well within the uncertainties (noted under the table) arising from reading and interpolating the curves of Fig. 4 of Ref. 3.

CONCLUDING REMARKS

It has been shown that the focus booms that arise in turning flight at supersonic speed can be suppressed by the simple expedient of slowing down the aircraft. The correct deceleration will eliminate the local curvature of the wave front (Mach 'cone') responsible for the focussing. The curvature (concave outward) is proportional to the component of resultant acceleration resolved along the normal to the wave front. In the proposed scheme the tangential deceleration component is adjusted to cancel out the centripetal acceleration component.

Horizontal turns of a prescribed limited sharpness are not of concern for the above maneuver: their focus booms will be cut off from reaching the ground by atmospheric refraction. The minimum turn radius (~ maximum acceleration) for focus cutoff is related herein in a simple fashion to the tabulated width of the sonic boom carpet for rectilinear flight, as a function of Mach number and altitude.

The tabulated results confirm some well-known generalizations. Thus at cruising altitude (11 km.) flight faster than M = 1.7 will permit quite sharp turns (> 0.45 g) without focussed booms reaching the ground. But at transonic speeds even very gentle turns - which might be inadvertent - will yield focussed booms: a sufficient centripetal acceleration is 0.09 g at M = 1.2, with the value decreasing as M approaches unity.

These observations seriously limit the practical utility of the proposed deceleration scheme for suppressing focus booms; the scheme is virtually inapplicable at the lower supersonic speeds where the need for suppression is the greatest. In particular, deceleration is called for in the face of the requirement for acceleration to get up to cruising speed.

REFERENCES

1. Rao, P. S. "Supersonic Bangs". Parts I and II, Aero. Quarterly, Vol.7, Part I pp.21-44, Part II pp.135-155 (1956).

2. Wanner, J.C.L., Vallee, J., Vivier, C., Thery, C. "Theoretical and Experimental Studies of the Focus of Sonic Booms", Paper presented at 2nd. Sonic Boom Symposium, Acoust. Soc. Ameri., Houston, 3-6 November, 1970. (to be published in J. Acoust. Soc. Amer .).

3. Kane, E. J., Palmer, T. V. "Meteorological Aspects of the Sonic Boom", Report prepared for Federal Aviation Agency by the Boeing Co., September, 1964. Clearinghouse for Fed. Sci. and Tech. Information, Document No. AD 610463.

4. Kane, E. J. "Some Effects of the Nonuniform Atmosphere on the Propagation of Sonic Booms", J. Acoust. Soc. Amer., Vol. 39, No.5, Part 2, pp.S26-S42, May 1966.

TABLE I. CONDITIONS FOR NO FOCUS (CUTOFF) AT GROUND

(Flight Altitude 11 km (36,000 ft))

M	M_g	$\cos^2\mu_g$	B CUTOFF WIDTH	SHARPEST PERMISSIBLE TURN: MINIMUM RADIUS: PREDICTED $R = B\sec^2\mu_g$	MINIMUM RADIUS: REF. 2[3]	MAXIMUM ACCEL.[4]: REF. 2[3]	BANK ANGLE: REF. 2[3]	CHARACTER
v/a	v/a_g	$1-M_g^{-2}$	REF. 3[1] mi.[2]	mi.	mi.	g's	degrees	
1.2	1.042	.0789	7.0	88.9	87.2	.09	5°	very gentle
1.4	1.218	.328	14.5	44.2	47.1	.23	13°	gentle
1.7	1.478	.541	18.5	35.1	35.6	.45	24°	noticeable
2.4	2.083	.769	22.5	28.3	29.8	1.07	47°	military
3.0	2.606	.851	24.0	28.6	28.3	1.73	60°	military
				← COMPARE →				

FOOTNOTES:

1. Fig. 4
2. These figures, read or interpolated from graphs, are uncertain to about ± 0.5 mi. There is a corresponding uncertainty in column 5, varying from about ± 6 mi. at M = 1.2 down to about ± 0.6 mi. at M = 3.0
3. Fig. 4.
4. Horizontal component.

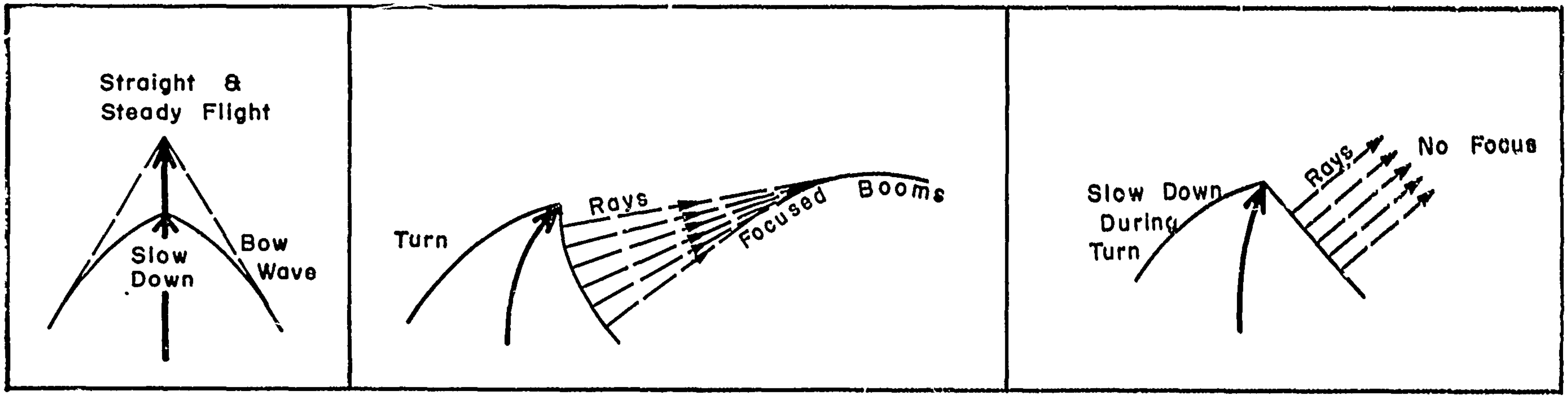

FIG. 1 BOW WAVE OF SUPERSONIC AIRCRAFT IS CONCAVE ON INSIDE OF TURN, WHICH LEADS TO A FOCUSED BOOM. SLOW-DOWN CURVES WAVE OPPOSITELY. COMBINED SLOW-DOWN & TURN YIELDS STRAIGHT BOW WAVE WITH NO FOCUS.

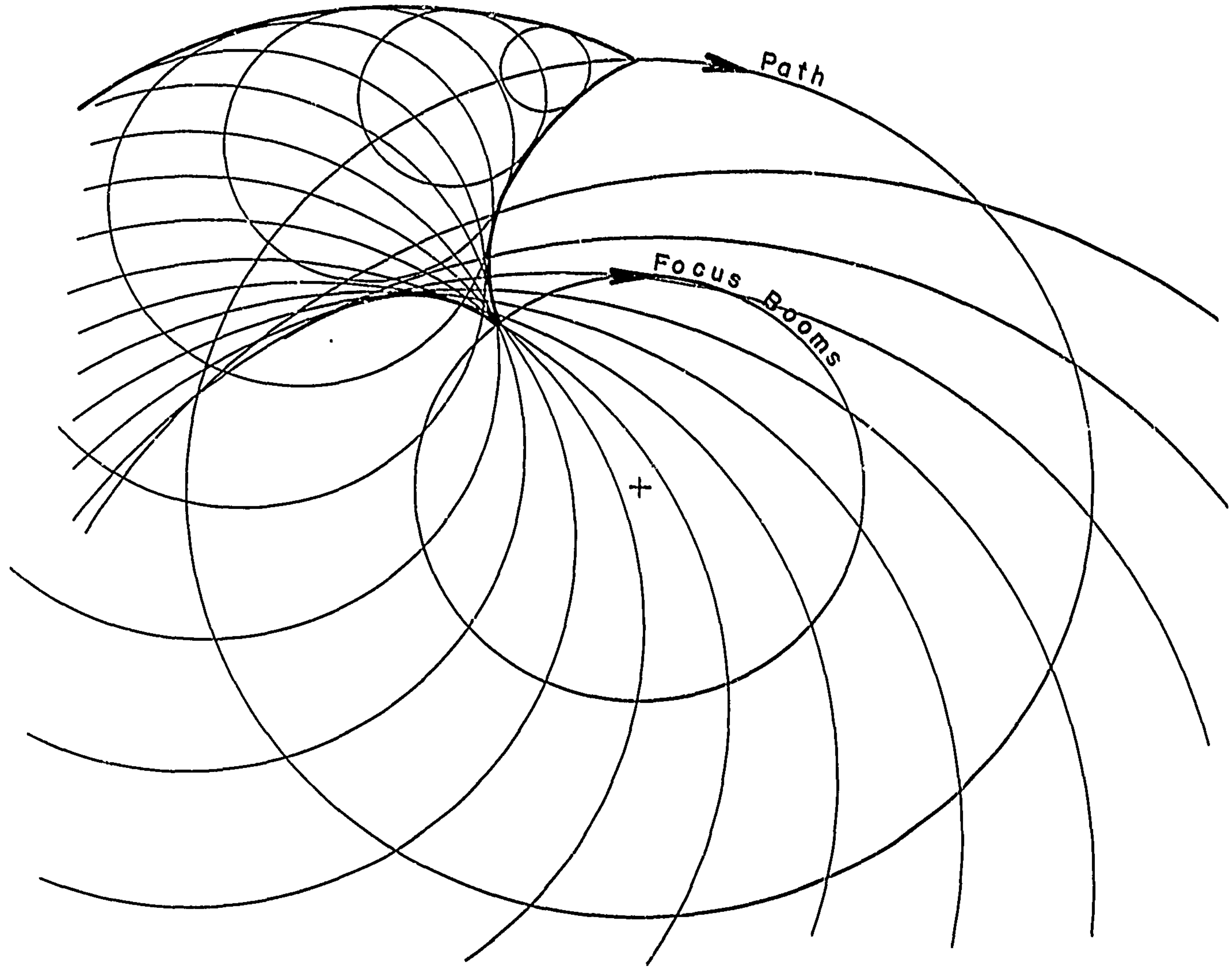

FIG. 2 FORMATION OF CUSPED WAVE ENVELOPE BY AIRCRAFT IN SUPERSONIC TURN

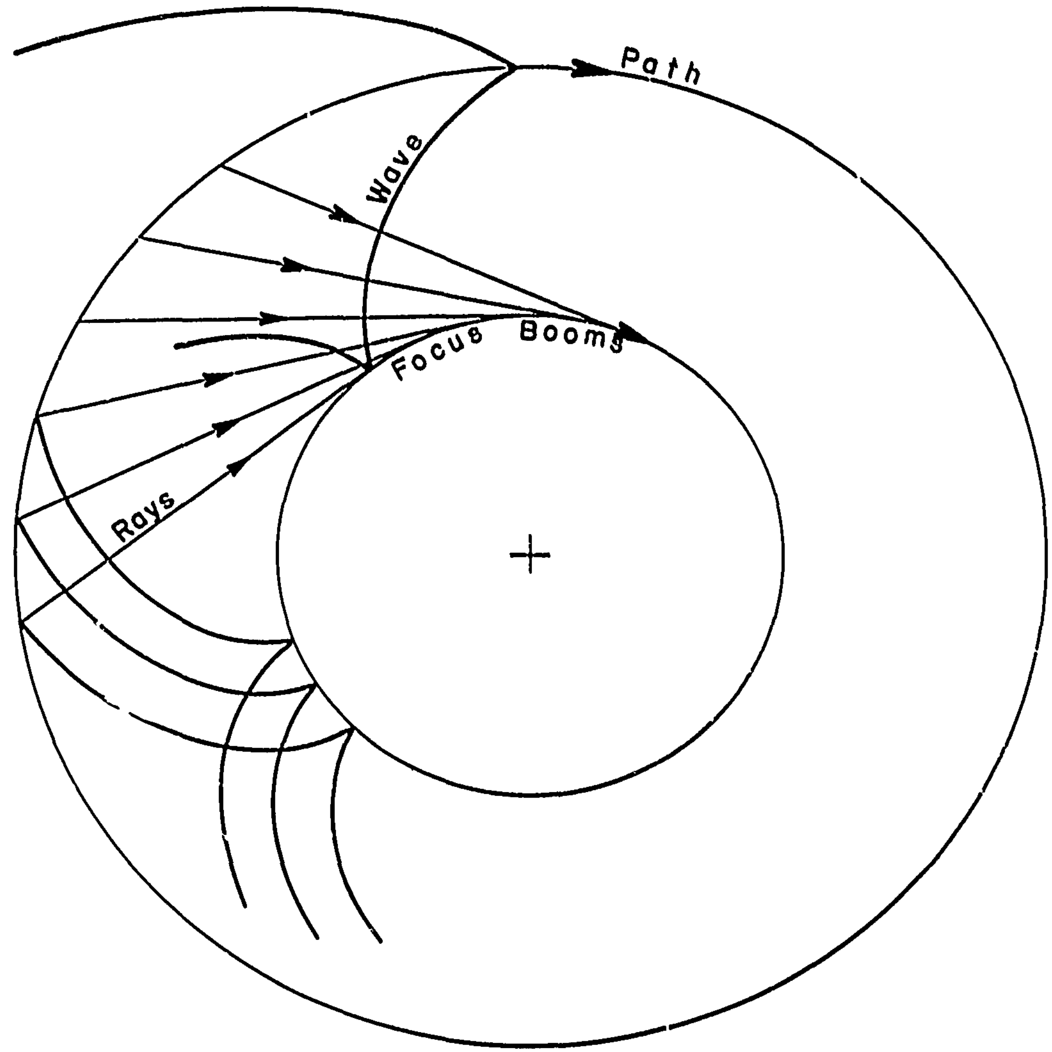

FIG.3 TIP OF CUSP CORRESPONDS TO POINT AT WHICH RAYS CONVERGE TO FOCUS

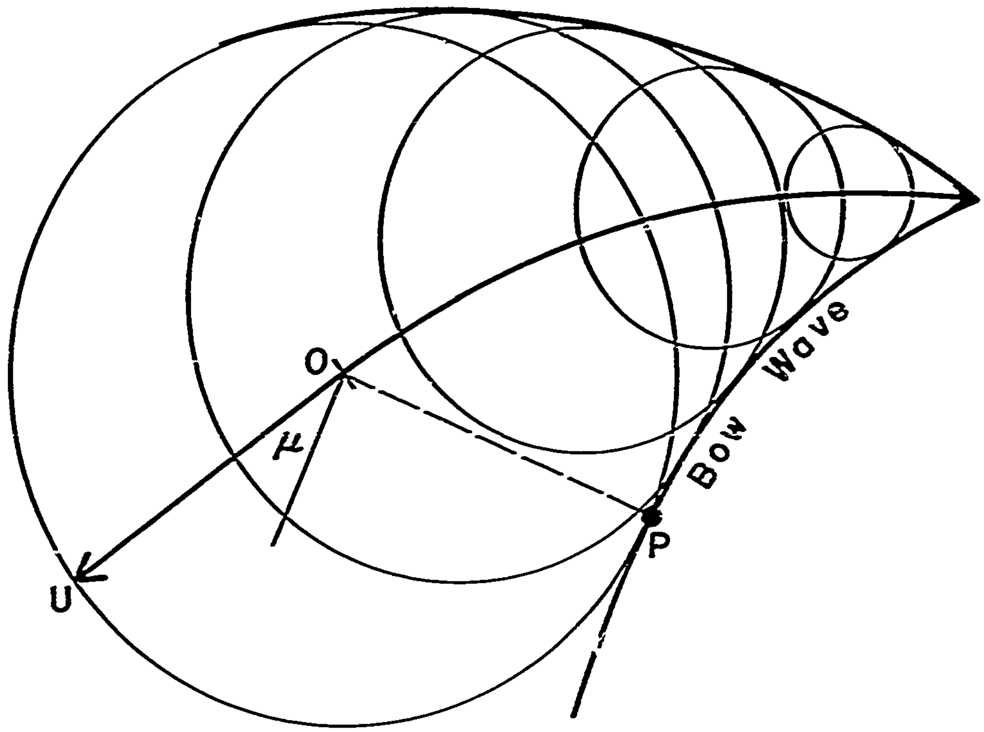

FIG. 4 O IS CENTER OF SOUND WAVE WHICH EXPANDS WITH TIME & RECEDES WITH FLIGHT SPEED U RELATIVE TO AIRCRAFT

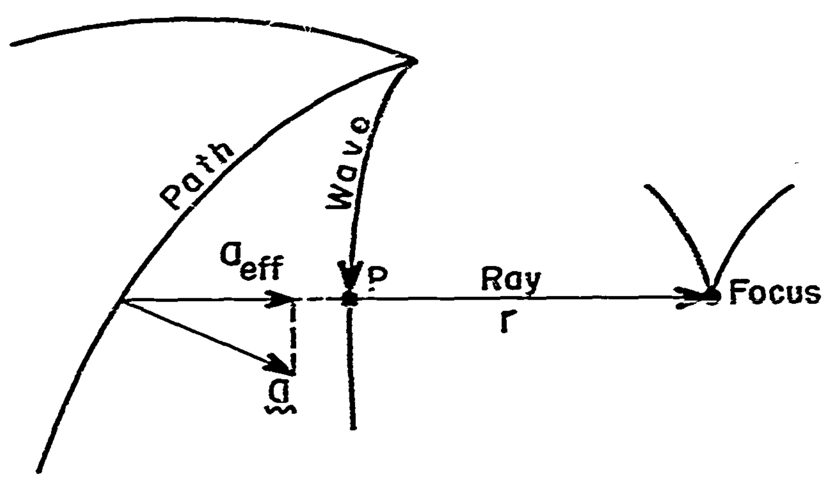

FIG. 5 P RUNS ALONG BOW WAVE WITH VELOCITY $U \cos\mu$ (FIG. 4) & ACCELERATION a_{eff} ; THESE DICTATE THE CURVATURE

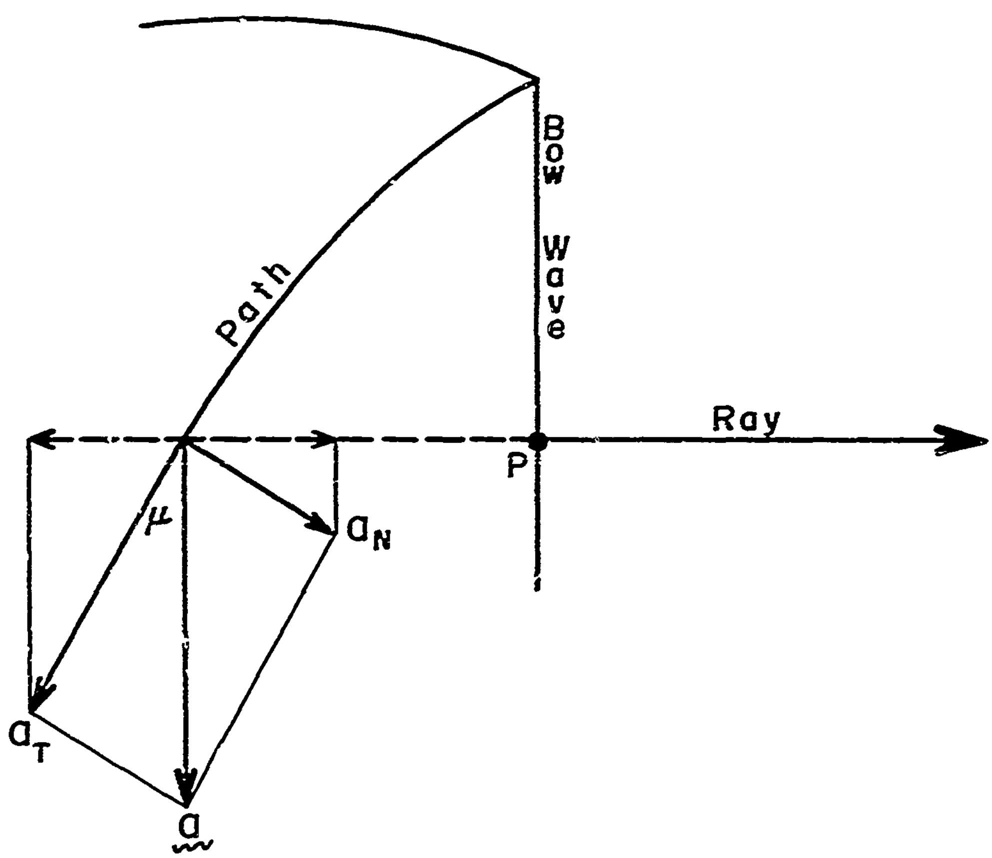

FIG. 6 TANGENTIAL DECELERATION ADJUSTED TO CANCEL EFFECT OF CENTRIPETAL ACCELERATION: BOW WAVE IS STRAIGHT IN PLANE OF TURN

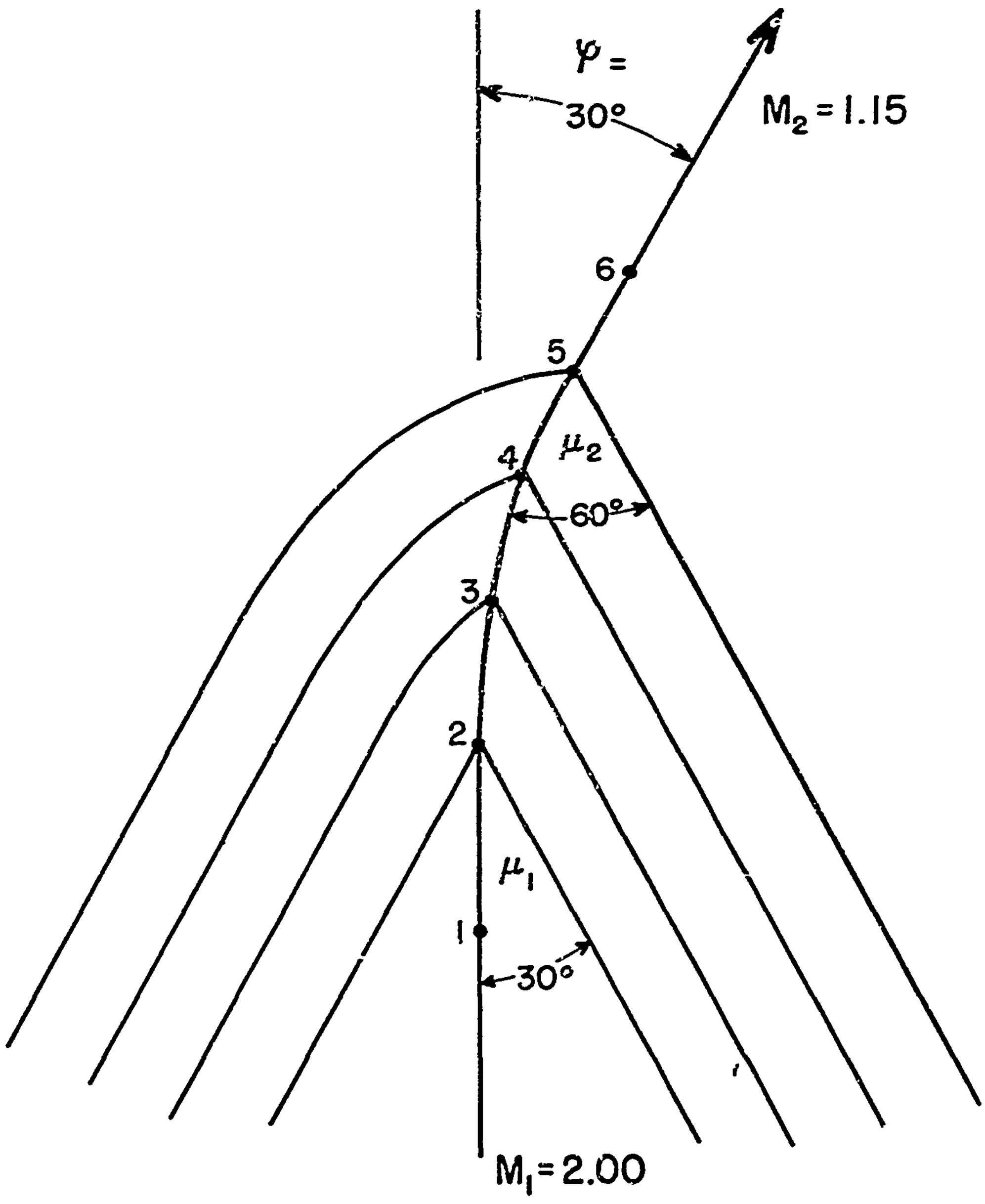

FIG. 7 EXAMPLE OF STRAIGHT-BOW-WAVE NO-FOCUS TURN

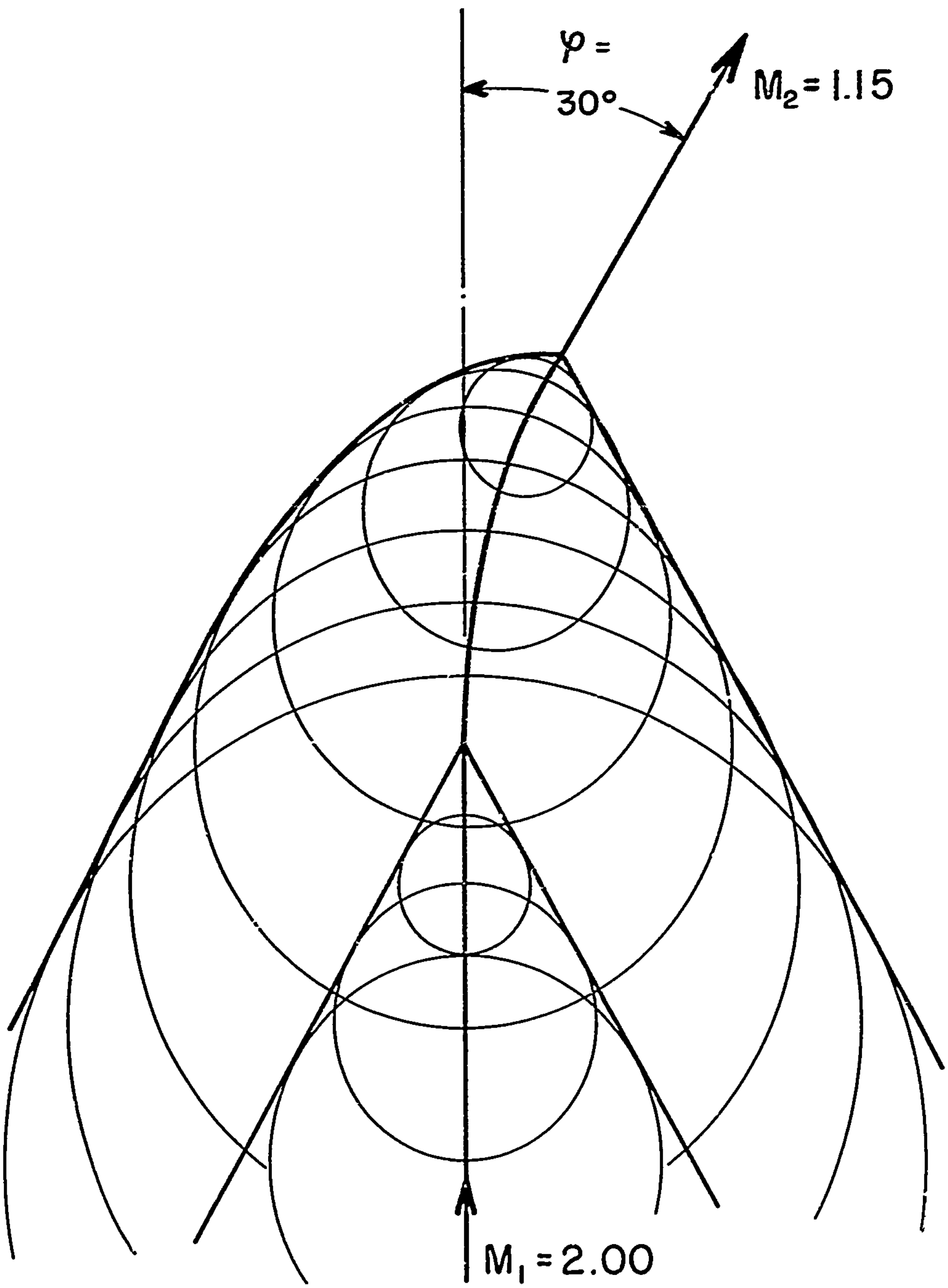

FIG. 8 EVOLUTION OF WAVE PATTERN MAINTAINING STRAIGHT BOW WAVE DURING NO-FOCUS TURN

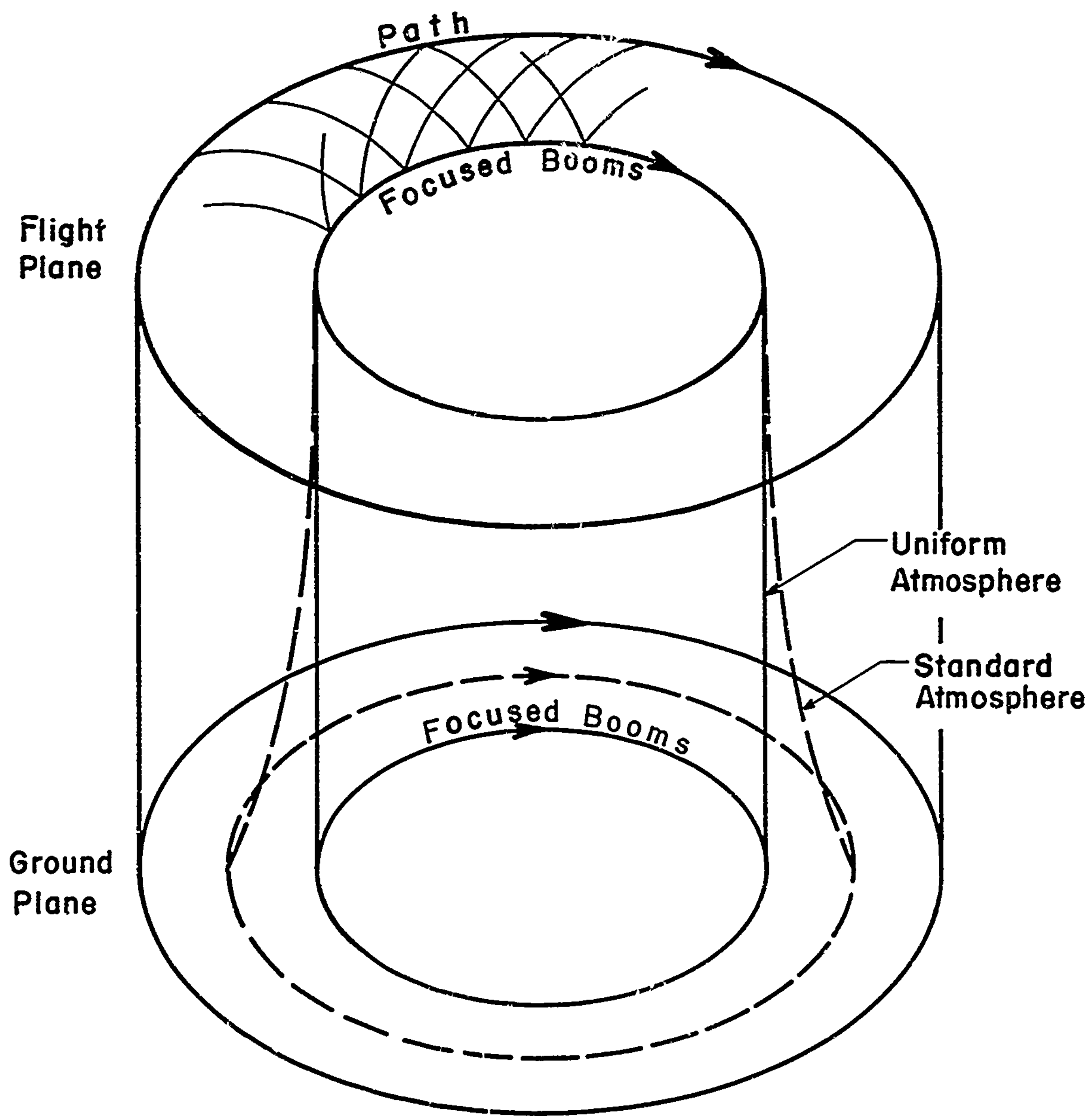

FIG. 9 LOCUS OF FOCUSED BOOMS IN THREE DIMENSIONS

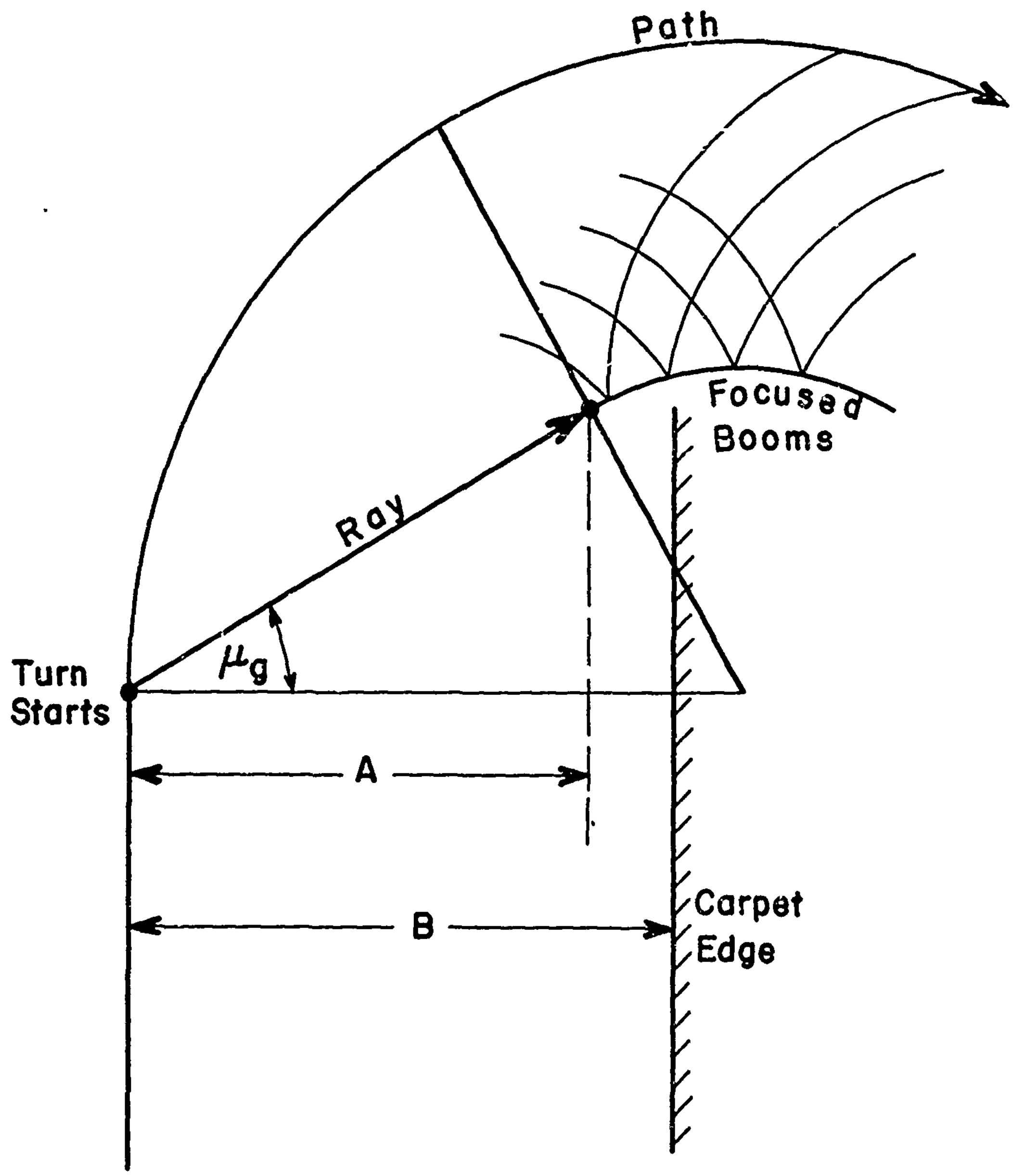

FIG. 10 REFRACTION CUT-OFF OF FOCUSED BOOMS OCCURS WHEN A > B

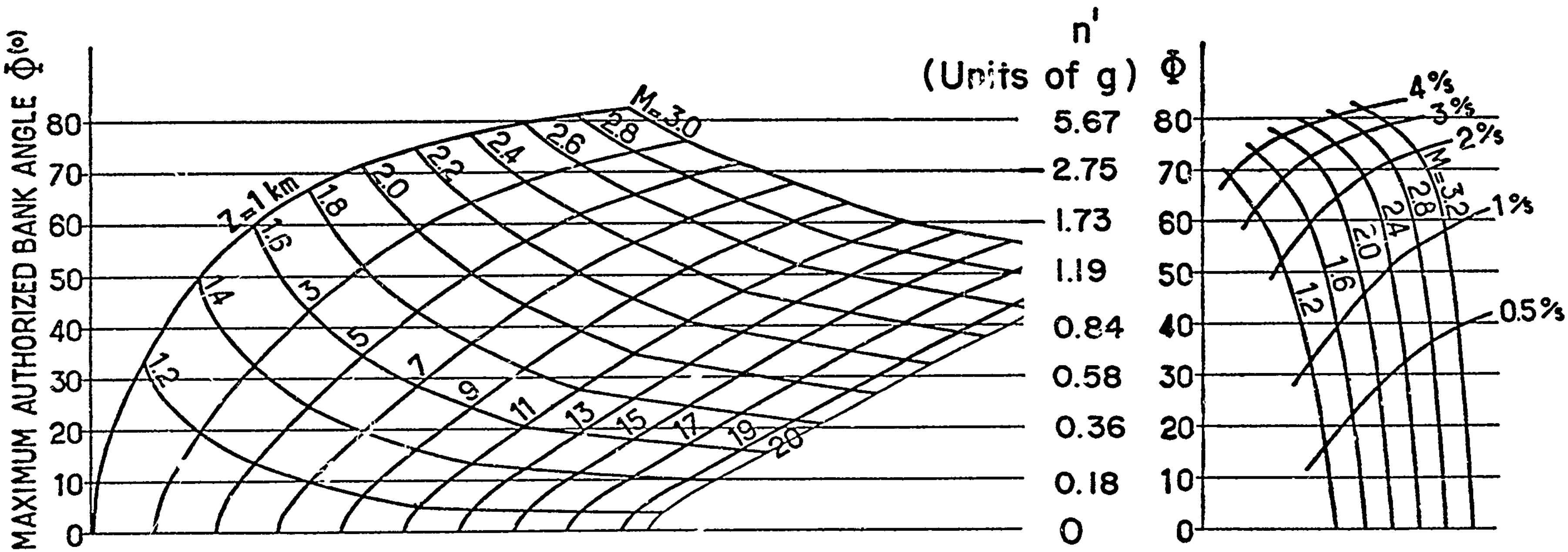

FIG. II CONDITIONS FOR NO-FOCUS (CUTOFF) AT GROUND FOR HORIZONTAL TURNS. LEFT HAND CHART GIVES MAXIMUM PERMISSIBLE BANK ANGLE. RIGHT HAND CHART GIVES CORRESPONDING RATE OF TURN REPRODUCED FROM FIG. 4 OF WANNER ET AL, REF. 2, WITH ADDED SCALE OF CENTRIPETAL ACCELERATION n'.